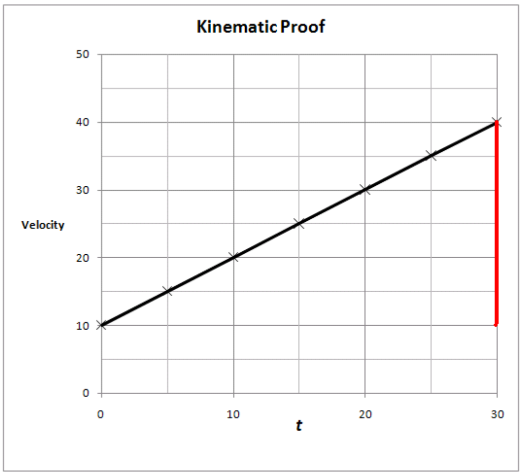
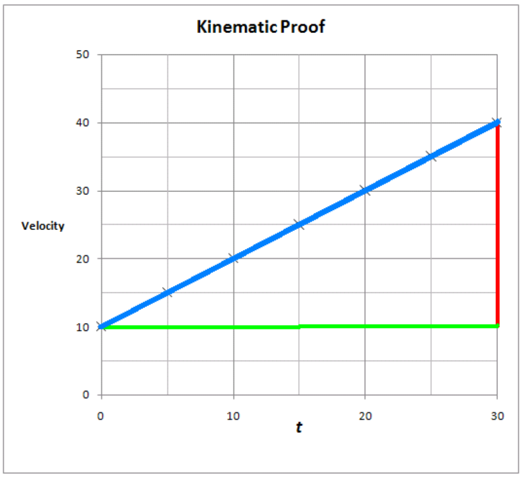
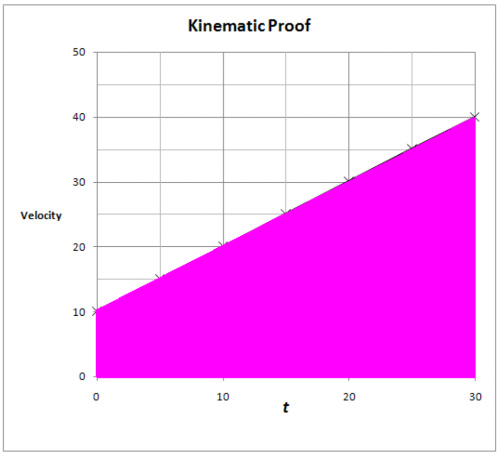
Kinematic Diagrammatic Proofs



Red = ΔV = Change in velocity = $v_f - v_i$



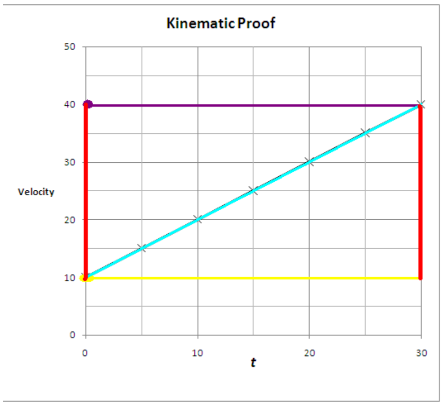
Blue = Acceleration= $a = \frac{\Delta V}{\Delta t} = \frac{v_f - v_i}{t}$



Area under graph = Distance=d

Mathematical Arrangements

Refer to the pictures for help



$$a = \frac{\Delta V}{t}$$

$$a = \frac{v_f - v_i}{t}$$

$$v_f - v_i = at$$

$$v_f = at + v_i$$

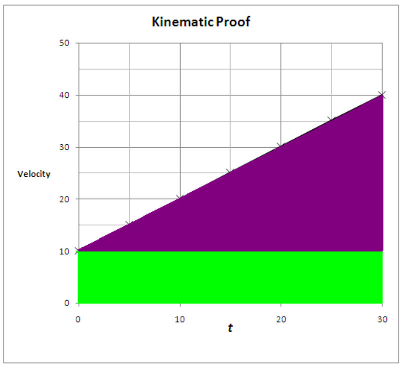
Kinematic Equation Number One

$$\boldsymbol{v_f = v_i + at}$$

Using the Area below the graph

Distance = Area under the graph

Method 1 – RST Rectangles Squares and Triangles



Area of a rectangle/ Square = Base X Height
Area of a triangle = $^1/_2$ Base X Height

Distance = Area of triangle + Area of Rectangle

Area of rectangle = B X H

$$Rectangle = v_i \times t$$
$$= v_i t$$

Area of triangle = ½ B X H

$$Triangle = \frac{1}{2} t \times \Delta v$$

$$Triangle = \frac{1}{2} t \times (v_f - v_i)$$

$$a = \frac{v_f - v_i}{t}$$

$$\rightarrow at = v_f - v_i$$

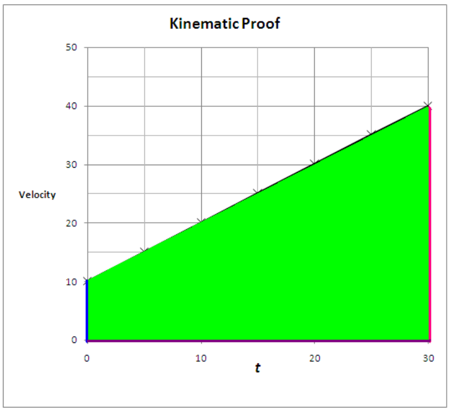
$$Triangle = \frac{1}{2} t \ \times at = \frac{1}{2} at^2$$

$$d = distance = Rectangle + Triangle = v_i t + \frac{1}{2} at^2$$

Kinematic Equation Number Two

$$\boldsymbol{d = v_i t + \frac{1}{2} at^2}$$

Method 2 – Trapeziums



Area of a trapezium =

$$\frac{(a+b)}{2} \times h$$

$a = v_i$

$b = v_f$

$h = t$

$$d = \frac{(a+b)}{2} \times h$$

Substitute in

$$d = \frac{(v_i + v_f)}{2} \times t$$

Kinematic Equation Number Three

$$d = \frac{v_i + v_f}{2} \times t$$

Mathematical Re-arrangement

To produce the fourth equation we use

$a = \frac{v_f - v_i}{t}$ Also written as $v_f = v_i + at$ **and** $d = \frac{v_i + v_f}{2} \times t$

$$\text{Re-arrange } a = \frac{v_f - v_i}{t}$$

$$\rightarrow at = v_f - v_i$$

$$\rightarrow t = \frac{v_f - v_i}{a}$$

$$\text{Substitute into } d = \frac{v_i + v_f}{2} \times t$$

$$d = \frac{v_i + v_f}{2} \times t$$

$$\rightarrow d = \frac{v_i + v_f}{2} \times \frac{v_f - v_i}{a}$$

$$\rightarrow 2ad = (v_i + v_f) \times (v_f - v_i)$$

$$\rightarrow 2ad = v_f^2 + v_i v_f - v_i v_f - v_i^2$$

$$\rightarrow 2ad = v_f^2 - v_i^2$$

$$\rightarrow v_i^2 + 2ad = v_f^2$$

$$v_f^2 = v_i^2 + 2ad$$

Kinematic Equation Number Four

$$v_f^2 = v_i^2 + 2ad$$